POTENTIAL AND FIELD COMPUTATIONS BY AN OPTIMIZED MONTE CARLO TECHNIQUE

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Abstract - The Monte Carlo method is applied to diverging field problems. The computational effort-expressed in this work by the number of steps in random walks-is related to the relative space potential, the prespecified walk termination distance and the degree of field nonuniformity in the gap. To obtain potential and field distributions in a given system, equations for these quantities are developed at neighbouring space points using Green's function. The accuracy of the algorithm is markedly enhanced by seeking the optimal spacing of those neighbouring points. The present technique is satisfactorily compared with the charge simulation method in a case study on hemispherically capped rod plane gap.

I. INTRODUCTION

Calculations of potentials or fields are needed for many problems in electrical power engineering as well as in heat conduction and fluid dynamics. The mathematical techniques needed are well known but are often difficult to apply in the complicated three-dimensional geometries that arise in practice. It is difficult to use finite element or finite difference methods to calculate electric potentials or fields near a high voltage transmission tower, or stress control fitting, because of the large number of mesh points or nodes needed to give an adequate representation of the geometry. The charge simulation method may likewise require the use of a large number of trial charges. The Monte Carlo method gives a convenient and flexible means of tackling these and similar problems in electrical power engineering [1,2,3] Mathematically, the Monte Carlo method solves the Dirichlet's problem by finding the potential and gradient which satisfies Laplace's equation within a given region and takes specified values on its boundary.

Basically, if the potential at a given point in space is required, a series of random walks is constructed starting at that point and reaching for a boundary where the potential is known. The average of the potentials gathered by this series of walks is a statistical estimate of the unknown potential of the point.

II. IMPLEMENTATION OF MONTE CARLO METHOD

The Monte Carlo method is implemented by initiating a series of random walks from the point in space where potential is to be evaluated. Each walk i will end when it reaches a boundary of a known potential V_{ri}. Based on N random walks the estimated potential of the point is

$$V_r = \frac{1}{N} \sum_{i=1}^{N} V_{r_i} \qquad (1)$$

In the floating random walk the length of the step is equal to the distance between the point and the nearest boundary. Random number generators determine the direction of the step.

The floating random walk procedure is applied to a three dimensional geometry by assuming a homogeneous spherical region with center at (x,y,z) and with radius r. The exact solution for the potential distribution $v(r,w,\phi)$ on the boundary based on the mean value theorem is as follows:

$$v(x,y,z) = \int_0^1 \int_0^1 v(r,w,\phi)\, dF\, dG \qquad (2)$$

In which $F = w/2\pi$, $G = 0.5(1-\cos\phi)$

where w and ϕ are, respectively, the polar coordinate angle and the cone angle in a spherical coordinate system. The functions F and G are probability distribution corresponding to w and ϕ.

A probabilistic interpretation of eqns. (2) indicates that a random walk instantaneously at (xi,yi, zi), will step to a new position on the circumference of the surrounding sphere in accordance with the probabilities F and G.As $N \to \infty$, the solution obtained from the floating random walk technique will approach the exact solution. A boundary is said to have been reached if the walk comes within a small distance H from it [4].

The potential V_p of a point at a position r_p is related to that of a point at r_o by [4]

$$V_p = \frac{1}{N} \sum_{i=1}^{N} w(r_p, r_o, r_i')\, V_{ri} \qquad (3)$$

where, r_i is the first step in the ith walk; V_{ri} as in eq. (1) is the potential of the boundary reached by the ith walk; w is a weight function derived from Green's function as [4]

$$w = \frac{a(a^2 - |r_p - r_o|^2)}{|r_p - r_i'|^3} \qquad (4)$$

where a is the distance between the initial point at r_o and the nearest boundary.

Therefore, the electric field is derived from eq. (3) to be

$$E_p = -\nabla V_p = \frac{-1}{N} \sum_{i=1}^{N} \nabla w.\, V_{ri} \qquad (5)$$

The present work shows that estimated potentials and fields at neighbouring points are sensitive to the distance between points r_o and r_p and optimal selection of that distance is, thus, called for.

III. SENSITIVITY ANALYSIS

The time and cost of computation using a monte carlo method depends directly on the product of the predetermined number of walks N by the average number of steps per walk S_w. The latter, in turn, depends on:-
(1) The relative potential itself at the location in question.

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(2) The degree of field nonuniformity.
(3) The walk termination distance H.

In order to investigate the effects of the above factors with certainty a standard analytically-solved configuration is chosen, namely, the case of coaxial cylinders.

The potential,at a point in a coaxial cylinder space is successively computed as the number of walks is increased. The rootmean square sampling error in the estimated potential V_r decreases as the number of walks is increased according to [5]

$$S = [\frac{1}{N} \sum_{i=1}^{N} (V_{r_i}^2 - V_r^2)/(N-1)]^{\frac{1}{2}} \qquad (6)$$

The effect of N is manifested in Fig. (1) plotted for a coaxial system of outer-to-inner radii ratio of ψ=50. The computed potentials are fluctuating about its known true value.

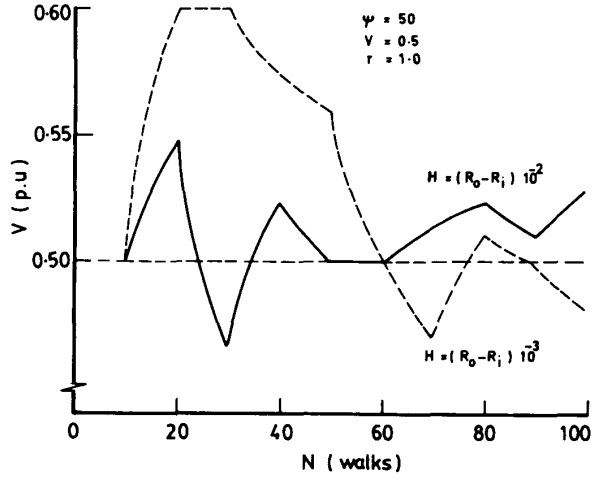

Fig. (1) Potential estimation by Monte Carlo

A. Relation of the Number of Steps Walk to Potential

The relation between the number of steps per walk Sw and the relative potential V is sought by applying the Monte Carlo algorithm at points of different relative potentials in the range O to 1.0. The average number of steps per walk was seen to be maximum at V=0.5 and decreases towards zero when V approaches the zero or 1.0 values. Based on the fact that the variance of a binomial distribution is $\theta(1-\theta)$ where θ is the probability of success [5], and bearing in mind that Sw reflects the"hesitation" in reaching a final boundary, the quantity Sw is assumed to be related to space potential by

$$Sw = K\, v(1 - v) \qquad (7)$$

where v is the relative space potential, and K is a constant. Several computer runs at v = 0.5 for various cases were made and the average Sw is used to compute K. The variation of Sw with space potential for different field nonuniformity (ψ) and different termination distance H are shown in Figs.(2) to (3).

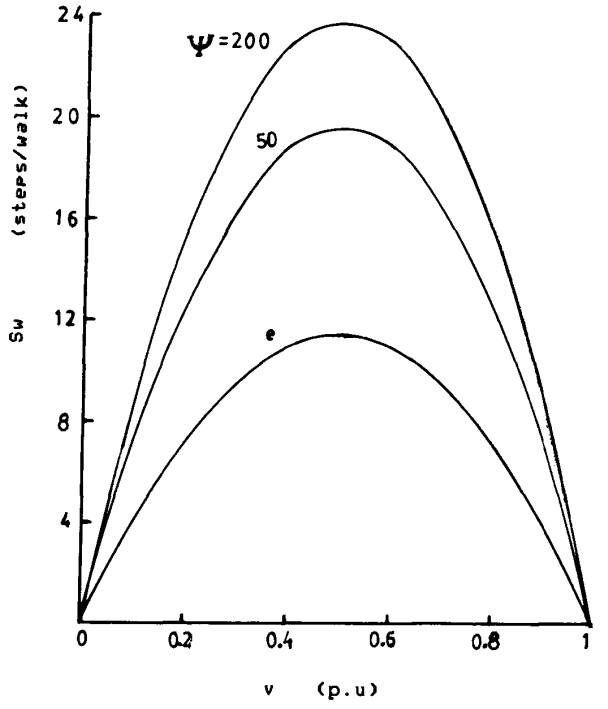

Fig. (2) Effect of potential and ψ on number of random steps

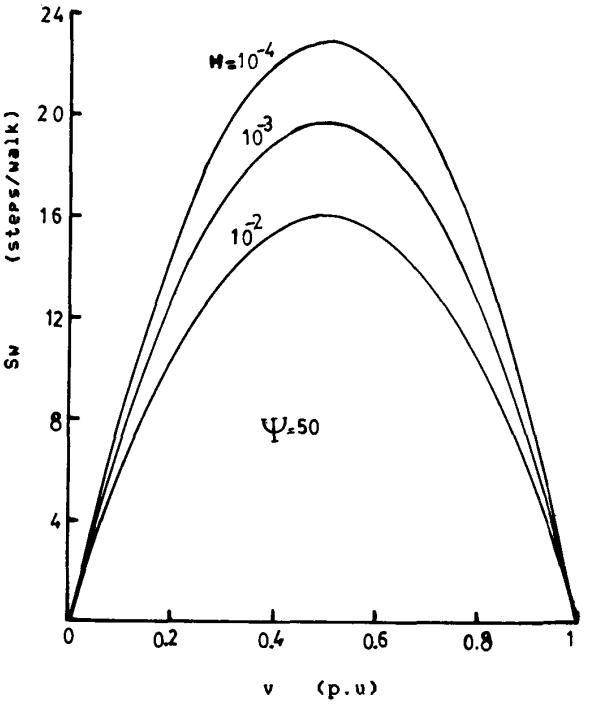

Fig. (3) Effect of potential and H on number of random steps

B. Relation of Sw to the Degree of Field Nonuniformity

The relation between the number of steps per walk Sw at a relative space potential of 0.5 (which is recognized as the maximum number of steps per walk Sw of a given case) and the ratio ψ could be fitted to the semi-logarithmic relationship:

$$Sw = a \log (\psi) + b \qquad (8)$$

It is seen in Fig. (4) that a is appoximately constant for different termination distances, and thus the relation between Sw and log (ψ) for various values of termination distances appear as parallel straight lines.

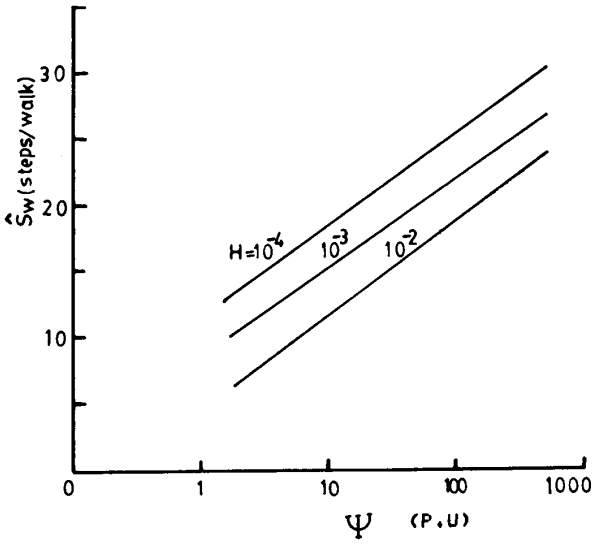

Fig. (4) Variation of number steps with field nonuniformity

C. Relation of Sw to the Termination Distance H

The relation between the maximum number of steps per walk Sw (occurring at a relative space potential of 0.5) and the termination distance H could also be fitted to the semi-logarithmic straight line equation.

$$Sw = c \log (H) + d \qquad (9)$$

It is noticed in Fig. (5) that c for different values of ψ is approximately unchanged while the constant d has a strong dependence on the degree of field non-uniformity ψ.

IV. OPTIMAL SELECTION OF NEIGHBOURING POINTS

The use of Green's function implies that if a series of random walks gives an estimate of a relative potential v at a point r it also contains information about the potentials at points in the neighbourhood of r, namely r_p as explained earlier in eqn. (3).

As r_p approaches the surface of the sphere of the first step in the walk the estimate V_p becomes dominated by a few random walks with large weighting factors and its accuracy falls off [4]. At distances r_p-r larger than half the radius of the first step the accuracy also diminishes markedly.

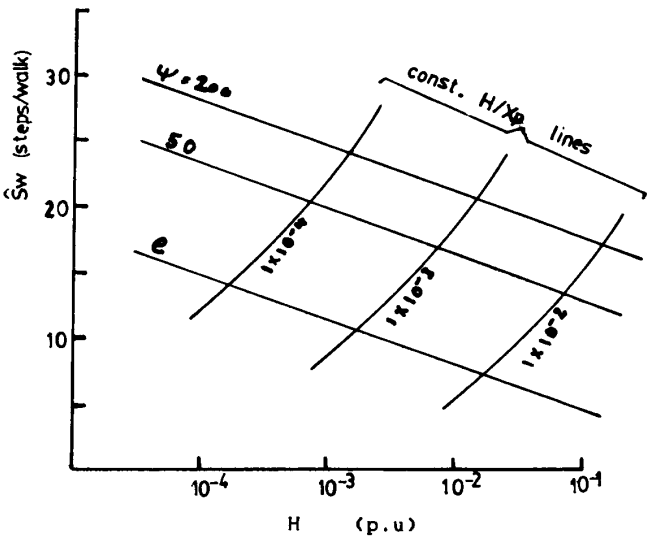

Fig. (5) Variation of number of steps with H.

In this work an attempt is made to detect the influence of the spacing between r and r_p on the accuracy of the potential estimation. This attempt will help determine the optimal choice of the sequence of points whenever the potential distribution along a particular line in space is required. Fig. (6) shows, for a case of known potential distribution, namely coaxial cylinders, the variation of the error in estimated relative space potential with the spacing Z = r_p-r. It appears that for sufficiently low spacing Z the potential error is large and negative (i.e. the estimated potential is less than its true value). The error decreases in magnitude until it vanishes in the shown case at Z = 15%. Thereafter, the error becomes positive and continues to increase with Z. The value of Z at which the potential error is zero is recorded for many cases.

Fig. (7) shows that the relation between the optimum distance Z_v and the relative space potential v is nearly linear over the potential range of 0.7-1.0 which is considered to be of prime importance to high voltage technology. At potentials below 0.7 the relationship becomes slightly nonlinear.

The above analysis was by necessity performed on an analytically solvable case namely the case of coaxial cylinders. The applicability of the computed optimal distances between neighbouring points is substantiated-towards the end of this paper-when a general non analytically solvable case is treated.

V. OPTIMAL DISTANCES FOR FIELD COMPUTATION

The electric field at any point in space can be evaluated according to eq. (5) which is based on the choice of a point r_p in the neighbourhood of the target point r. This choice is also found to influence the accuracy of the computed field significantly Fig. (8) illustrates-for a typical case-how the above choice of distance Z affects the error in the computed field.

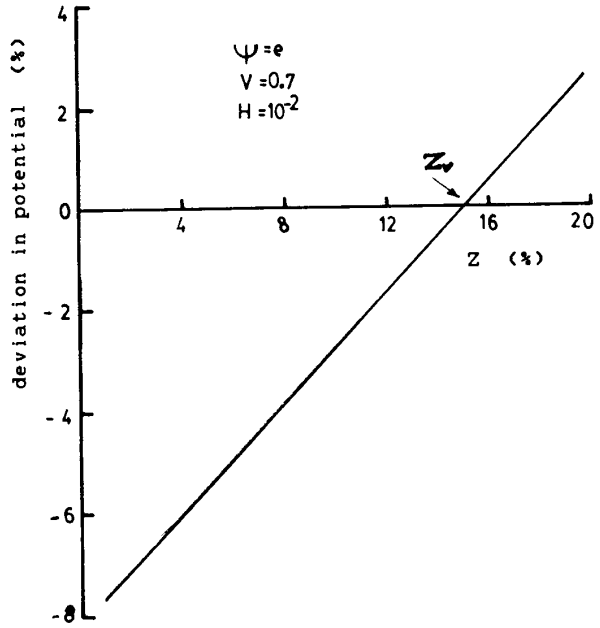

Fig. (6) Computed potential error

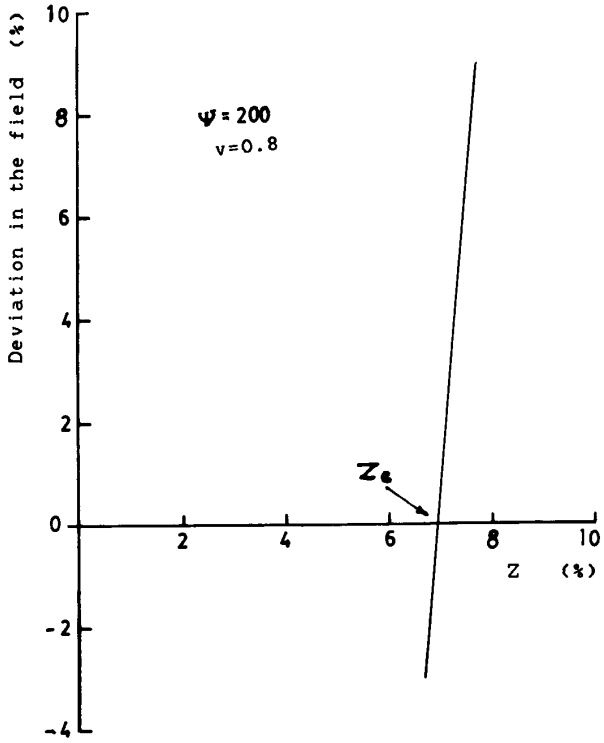

Fig. (8) Computed field error

Based on the standard case of coaxial cylinder several cases were run to estimate the optimal distance Z_e. The results are shown in Fig. (9) where Z_e is expressed as a percentage of the length of the first step in a random walk. It is noticed that as the level of field non uniformity increases optimal distances tend to decrease. On the boundary when the potential is unity the Monte Carlo method fails to estimate the field. This is manifested in Fig. (9)-at V=1- by calling for Z_e to be equal to zero a fact which renders Green's function impossible to apply.

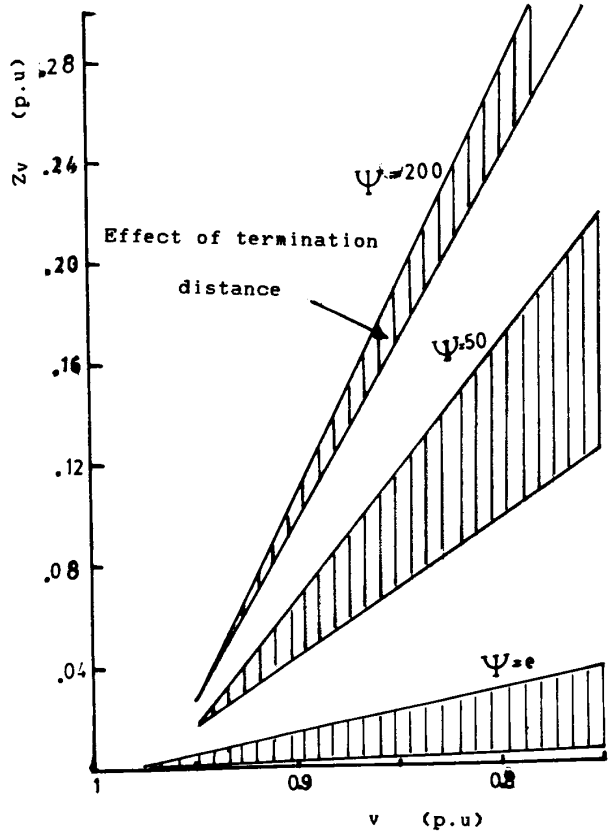

Fig. (7) Optimal distance for potential estimation

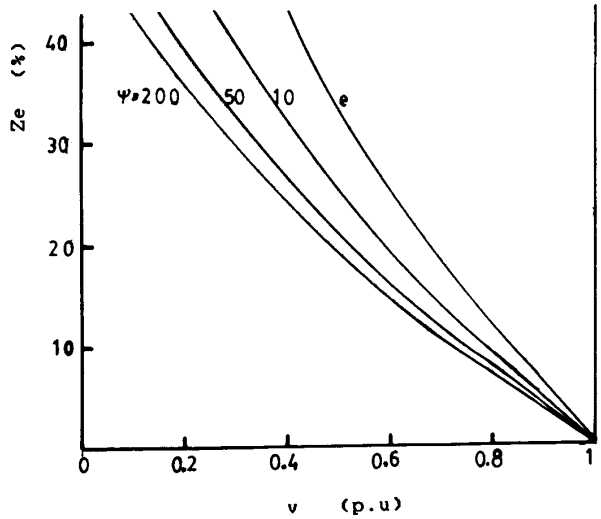

Fig. (9) Optimal distance for field estimation

VI. STUDY CASE

It is meant to verify the ability of the Monte Carlo method, in general, and the presently developed improvements, in particular, to handle a typical high voltage problem. The rod-to-plane gap, one of the most basic arrangements in high voltage research, is chosen for this purpose. This non-uniform field configuration has been a suitable tool for experimental observation due to the local confinement of prebreakdown ionization around the stressed electrode. Field knowledge will certainly help explain many gaseous ionization phenomena and elucidate the processes of gas breakdown. The solution is compared to those given by the Charge simulation method and also the tank model experimental technique.

A. Gap Arrangement

The rod-to-plane gap is seen in Fig. (10). The rod electrode is standardized as a cylindrical shaft with a hemispherical tip of equal diameter of 2.

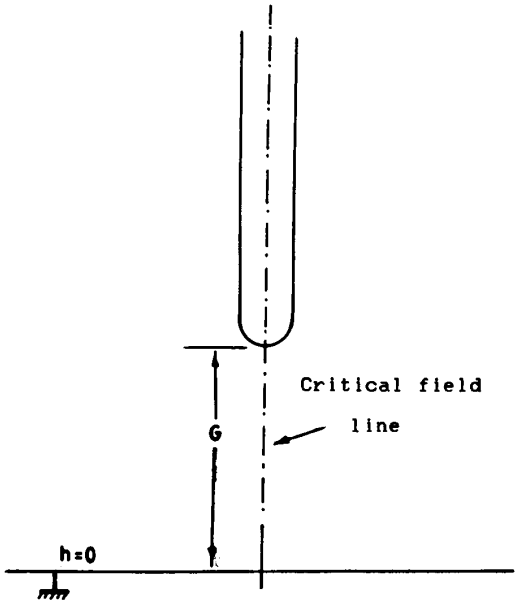

Fig. (10) rod-to-plane gap

Table I The potential and field distributions on the critical field line for G = 200

Distance from plane	Charge simulation ref.[6]		improved Monte Carlo method	
	v(p.u)	Ep(p.u)	v(p.u)	Ep(p.u)
199.9	0.9463	0.4884	0.9467	0.4981
199.8	0.9015	0.4118	0.9343	0.4119
199.7	0.8634	0.3528	0.8657	0.3517
199.6	0.8305	0.3064	0.8514	0.3040
199.5	0.8018	0.2691	0.7946	0.2673
199.4	0.7765	0.2387	0.8019	0.2359
199.3	0.7539	0.2136	0.7612	0.2115
199.2	0.7336	0.1927	0.7437	0.1904
199.1	0.7157	0.1749	0.7287	0.1750
199.0	0.6985	0.1597	0.7101	0.1601

Table II Comparison of the potential calculated by three methods. For G=160

Distance from plane	Charge simulation ref. [6]	improved Monte Carlo method	Tank model
159.9	0.9453	0.9567	0.933
159.8	0.8996	0.9133	0.897
159.7	0.8607	0.8567	-----
159.6	0.8272	0.8567	-----
159.5	0.7980	0.8033	0.824
159.4	0.7721	0.7667	-----
159.3	0.7491	0.7700	-----
159.2	0.7285	0.7267	-----
159.1	0.7097	0.7200	-----
159.0	0.6927	0.7167	0.725
158.0	0.5779	0.5733	-----
157.0	0.5125	0.5233	0.556
156.0	0.4680	0.4967	-----
155.0	0.4349	0.4400	0.483

The Charge simulation method was applied to the study arrangement by Abou-Seada and Nasser [6]. They used one point charge located at the center of the hemispherical portion of the boundary and a nine semi-infinite line charges located along the axis of the cylindrical portion. Also seven boundary points were selected.

Meanwhile, a computer program is written which applies the principles and recommendations reached in this paper to improve the Monte Carlo application. A floating random walk procedure was followed in which the optimal spacing among neighbouring points as determined above is put to use.

In Table I it is shown that the results of the Monte Carlo method are very close to those obtained by the Charge simulation method. The computing effort made during the present Monte Carlo application is relatively very small. Typically, the determination of the potential and field at any one point-near the rod tip-required a total of 100 walks equivalent to about 2 seconds on a VAX-11/780 computer.

In Table II a comparison is made among the results taken from the Charge simulation method, the present approach, and those measured experimentally using a tank model for a gap ratio of 160 [6]. It appears that the results obtained with the present technique are even closer to those of the tank model than the results of the Charge simulation method.

VII. CONCLUSIONS

(1) The computational effort expressed primarily by the number of steps per walk is found to be strongly dependent on several factors, namely, the relative space potential at the location in question, the degree of field nonuniformity and walk termination distance.

(2) The maximum number of steps per walk is encountered at a relative space potential of 0.5 and decreases towards zero when the potential approaches zero or 1 values.

(3) The maximum number of steps per walk increases linearly with the logarithm of the field nonuniformity factor.

(4) The maximum number of steps per walk increases linearly with the logarithm of the termination distance.

(5) The use of Green's function gives direct estimates of fields and allows each random walk to contribute directly to estimates of the potential and field at more than one point.

(6) The error in the estimated potential and field of a neighbouring point is minimum at some optimal distance between the target point ro and that neighbouring point rp.

(7) The optimality results obtained with coaxial systems are found to be very well applicable in other, non-analytically solvable, three-dimensional systems such as a rod-plane gap.

(8) Excellent agreement is obtained when the results of the present technique are compared to those of the charge-simulation method as applied to a hemispherically capped rod-plane gap.

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